**Abstraction step**

Let x = number of chairs to be produced by the manufacturer

y = number of tables to be produced by the manufacturer

Z = Total profit made from the above product mix

**Objective function, Z**

Total profit, Z = 45x + 80y

Max Z = 45x + 80y

**Constraints**

Mahogany timber restriction: 5x + 20y ≤ 400 or x + 4y ≤ 80

Labour hours restriction: 10x + 15y ≤ 450 or 2x + 3y ≤ 90

**Non negativity condition**

x, y ≥ 0

**LP model in summary**

Max Z = 45x + 80y

S.t

x + 4y ≤ 80

2x + 3y ≤ 90

x, y ≥ 0

**Standard form of the LP model**

Max Z - 45x -80y +0s1 +0s2

S.t

x + 4y +s1 = 80

2x + 3y +s2 = 90

x, y,s1,s2 ≥ 0

**Initial simplex tableau**

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| Basis | x | y | S1 | S2 | RHS | Ratios |
| S1 | 1 | **4** | 1 | 0 | 80 | 20 |
| S2 | 2 | 3 | 0 | 1 | 90 | 30 |
| Z | -45 | -80 | 0 | 0 | 0 |  |

**Initial solution**

x = y = 0 and s1 = 80, s2 = 90, Z = 0

The solution is not optimal as there are negative coefficients in the Z-row.

Entering variable that give the pivot column is y

Leaving variable that give the least positive ratio = s1

Pivot element = 4

**Improved simplex tableau 1**

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| Basis | x | y | S1 | S2 | RHS | Ratios |
| y | 1/4 | 1 | 1/4 | 0 | 20 | 80 |
| S2 | **5/4** | 0 | -3/4 | 1 | 30 | 24 |
| Z | -25 | 0 | 20 | 0 | 1600 |  |

**Improved solution 1**

x = 0, y =20 and s1 = 0, s2 = 30, Z = 1600

The solution is not optimal as there is a negative coefficient in the Z-row.

Entering variable that give the pivot column is x

Leaving variable that give the least positive ratio = s2

Pivot element = 5/4

**Improved simplex tableau 2**

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| Basis | x | y | S1 | S2 | RHS |  |
| y | 0 | 1 | 2/5 | -1/5 | 14 |  |
| x | 1 | 0 | -3/5 | 4/5 | 24 |  |
| Z | 0 | 0 | 1 | 4 | 2200 |  |

**Improved solution 2**

x = 24, y =14 and s1 = 0, s2 = 0, Z = 2200

The solution is optimal since there is no negative coefficient in the Z-row.

**Interpretation or advice to the manufacturer**

The manufacturer should produce 24 chairs and 14 tables so as to maximize his profit to Ksh. 2200. If the manufacturer could get an extra metre of Mahogany timber then it would have a value Sh.1 and one extra hour of labour would have a value of Sh. 4. [Shadow prices]

**Assignment 2**

1. For the following primal LP model find the Dual LP model and solve the Primal LP model and read the solution to both models from the final primal tableau.

Max Z = 14x + 7y

S.t

2x + 5y ≤ 18

5x + 2y ≤ 24

x, y ≥ 0

1. For the following primal LP model find the Dual LP model and solve the Dual LP model and read the solution to both models from the final dual tableau.

Min Z = 3x1 +9x2

S.t

2x1 + x2 ≥8

x1 + 2x2 ≥ 8

x1, x2 ≥ 0